

GOVERNMENT OF WEST BENGAL

NTSE 2013

SAT

HINTS & SOLUTIONS

MATHEMTICS

1. $x + \frac{1}{y} = 1 \dots\dots\dots(1)$

$y - \frac{1}{z} = 1 \dots\dots\dots(2)$

$xy + 1 = y$

Put, y in (2)

$xy + 1 - \frac{1}{x} = 1$

$Xyz = 1$

2. 40% discount on Rs.1000 = Rs.400 ----- (1)

Equivalent discount of successive discounts 30% & 10% is

$\left(30 + 10 - \frac{30 \times 10}{100}\right) \%$

37%

37% discount on Rs.1000 = Rs.370 ----- (2)

(1) — (2) = Rs.30

3. Circum radius $R = \frac{9}{\sqrt{3}}$

In Radius = $r = \frac{9}{2\sqrt{3}} = \frac{f \frac{a^2}{3}}{f \frac{a^2}{4.3}} = \frac{4}{1}$

4. $(x^2 - 1)^2 + (2 - x)^2 = (x^2 + 1)^2$

$$a^2 + b^2 = c^2$$

Right angled Triangle

5. $20 \div 5 \times 6 + 2 - 10$

$$100 - 3 + 10 = 107$$

6. $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$

$$= \sqrt{\sec^2 \theta - 1 + \operatorname{cosec}^2 \theta - 1 + 2}$$

$$= \sqrt{\tan^2 \theta + \cot^2 \theta + 2}$$

$$= \sqrt{(\tan \theta + \cot \theta)^2}$$

$$= \tan \theta + \cot \theta$$

7. Let length of wire be = l

$$4a = l$$

$$a = \frac{l}{4}$$

$$2\pi r = l$$

Circle is bigger

8. $\angle A = 65^\circ$; $\angle B = 55^\circ$
 $\angle C = 115^\circ$
 $\angle D = 125^\circ$

9. Volume of small cone = $\frac{1}{27}$ volume of big cone

10. $P = k w^2$

Let 1 kg diamond cost be Rs.2000
then

$$2000 = k \cdot 1^2 = k = 2000$$

$$P = 7500 \quad \text{Price of } \frac{1}{2} \text{ kg is } 7500 \quad \text{Loss} = 2000 - (2 \times 500) = \text{Rs.1000}$$

11. S.I. C.I,
 $r = 5\%$ $r = 10\%$
 $n = 3$ $n = 3$
 $S.I. = \frac{3P}{20}$ $C.I. = P \left(1 + \frac{10}{100}\right)^3 - P$
 $C.I - S.I. = 905.$
 $P = \text{Rs.}5000$

12. $2\pi r = S$ $\pi r^2 = A$
 $4\pi r^2 = S^2$
 $4\pi (\pi r^2) = S^2$
 $4\pi A = S^2$
 $\therefore S^2 = 4\pi A$

13. Given $\tan 9^\circ = \frac{x}{y}$
 $\frac{\sec^2 81^\circ}{1 + \cot^2 81^\circ} = \frac{\sec^2 81^\circ}{\csc^2 81^\circ}$
 $= \tan^2 81^\circ$
 $= \cot^2 9^\circ$
 $= \frac{1}{\tan^2 9^\circ}$
 $= \frac{y^2}{x^2}$

14.

Given

$$x^2 + y^2 + 10 = 2\sqrt{2}x + 4\sqrt{2}y$$

$$x^2 - 2\sqrt{2}x + y^2 - 4\sqrt{2}y + 10 = 0$$

$$x^2 - 2 \cdot x \cdot \sqrt{2} + (\sqrt{2})^2 + y^2 - 2 \cdot y \cdot 2\sqrt{2} + (2\sqrt{2})^2 = 0$$

$$(x - \sqrt{2})^2 + (y - 2\sqrt{2})^2 = 0$$

$$x - \sqrt{2} = 0 \quad \text{and} \quad y - 2\sqrt{2} = 0$$

$$x = \sqrt{2} \quad \text{and} \quad y = 2\sqrt{2}$$

$$\therefore x + y = 3\sqrt{2}$$

15.

$$x^2 - 6x + 5 < 0$$

$$x^2 - x - 5x + 5 < 0$$

$$(x-1)(x-5) < 0$$

$$\therefore 1 < x < 5$$

16.

Let ^{marked} Actual C.P. = Rs 125
~~C.P.~~ Price paid by the businessman

$$= 80\% \text{ of } 125$$

Profit for him at the time
of buying = 20% of 125
= Rs 25

S.P. ~~C.P.~~ of the businessman
= 120% of Rs 125

Profit for him at the time
of selling = 20% of 125
= Rs 25

Total profit = Rs 50

$$\text{Profit \%} = \frac{50}{125} \times 100$$

$$= 40\%$$

17.

7) Let 'abc' be 3 digit number

$$abc = 100a + 10b + c$$

When $(a+b+c)$ is subtracted from $(100a+10b+c)$, then

$$\text{the result} = (100a+10b+c) - (a+b+c)$$

$$= 99a + 9b$$

$$= 9(11a+b)$$

\therefore The result is divisible by '9'

18.

$$x = a \cos^3 \theta \quad \left| \quad y = a \sin^3 \theta \right.$$

$$\cos \theta = \left(\frac{x}{a}\right)^{1/3} \quad \left| \quad \sin \theta = \left(\frac{y}{a}\right)^{1/3} \right.$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\left[\left(\frac{x}{a}\right)^{1/3}\right]^2 + \left[\left(\frac{y}{a}\right)^{1/3}\right]^2 = 1$$

$$\left[\therefore x^{2/3} + y^{2/3} = a^{2/3}\right]$$

19.

$$\begin{aligned} \text{ar (Square ABCD)} &= 2 \times 2 \\ &= 4 \text{ sq. cm} \end{aligned}$$

$$\begin{aligned} \text{ar (Sector APS)} &= \frac{1}{4} \times \pi \times 1^2 \\ &= \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} \text{ar (Sector BPQ)} &= \text{ar (Sector CQR)} \\ &= \text{ar (Sector DRS)} = \frac{\pi}{4} \end{aligned}$$

area of non shaded region

$$\begin{aligned} &= 4 \times \frac{\pi}{4} \\ &= \pi \text{ sq. cm} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of the shaded region} &= \text{Area of Square ABCD} - \\ &\text{area of the non shaded region} \\ &= (4 - \pi) \text{ sq. cm} \end{aligned}$$

20.

$$\begin{aligned} A &= \sin^2 \theta + \cos^4 \theta \\ &= \sin^2 \theta + \cos^2 \theta \cdot \cos^2 \theta \\ &= \sin^2 \theta + \cos^2 \theta (1 - \sin^2 \theta) \\ &= \sin^2 \theta + \cos^2 \theta - \sin^2 \theta \cos^2 \theta \\ &= 1 - \frac{1}{4} (2 \sin \theta \cos \theta)^2 \\ &= 1 - \frac{1}{4} \sin^2 2\theta \end{aligned}$$

$$\text{Min} (\sin^2 2\theta) = 0$$

$$\text{Max} (\sin^2 2\theta) = 1$$

$$\begin{aligned} \text{Min} (A) &= 1 - \frac{1}{4} \cdot \text{Max} (\sin^2 2\theta) \\ &= 1 - \frac{1}{4} \cdot 1 = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{Max} (A) &= 1 - \frac{1}{4} \cdot \text{Min} (\sin^2 2\theta) \\ &= 1 - \frac{1}{4} \cdot 0 = 1 \end{aligned}$$

$$\therefore \frac{3}{4} \leq A \leq 1$$

PHYSICS:

21. [Power] = [force X velocity]

$$= [MLT^{-2}] [LT^{-1}] = ML^2T^{-3}$$

23. $p = \sqrt{2Em} \therefore \frac{p_1}{p_2} = \sqrt{\frac{E_1 \times m_1}{E_2 \times m_2}} = \sqrt{\frac{1}{2} \times \frac{1}{2}} = 1:2$

24. $a = \frac{F}{M_1 + M_2}$, $N = M_2 \frac{F}{M_1 + M_2} = \sqrt{\frac{1}{2} \times \frac{1}{2}} = 1:2$

25. $\frac{x}{5} = \frac{2x - 32}{9}$

$$x = 160$$

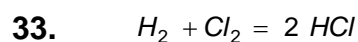
28. $n = \frac{v}{\lambda} = \frac{3 \times 10^8}{5000 \times 10^{-10}} = 6 \times 10^{14} \text{ Hz}$

29. \vec{E}_1 at 5 cm = $k \frac{5Q}{5^2}$

$$= 4 : 1$$

30. $(2+3) \Omega \parallel (4+6) \Omega = \frac{10}{3} \Omega$

CHEMISTRY:



$$2 \quad 1.2$$

$$.8 \quad 0 \quad 2.4$$

Here Cl_2 is the limiting reagent. After the reaction the composition by vol. of the resultant mixture will be 0.8 lit of hydrogen & 2.24 lit of hydrogen chloride.

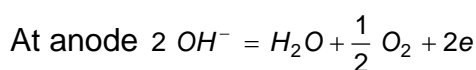
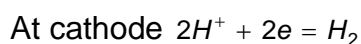
34. Higher the oxidation state of central atom higher the acidity.

35. $(1.88 - 1.35) \text{ gm} = 0.53 \text{ gm}$

0.53 gm of oxygen reacts with 1.35 gm of 'X'

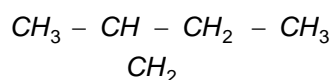
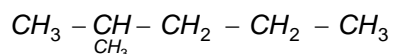
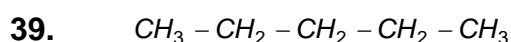
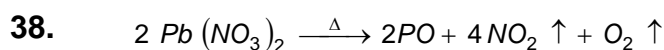
$$16 \text{ gram oxygen with } \frac{1.35 \times 16}{0.53} = 40.75$$

36. Liq. NH_3



At medium NaOH will be present and $\text{pH} > 7$

So, colour of the solution will turn yellow.



40. Oxygen is highly reactive that's why reaction with oxygen forms its oxide and can not be used to extract a metal from its ore.

41. Radioactivity is a nuclear phenomenon that's why it is not a periodic property.

